PROPOSTA RESOLUÇÃO PROVA 1 | 2022

António Leite

1.

1.1. O ponto B pertence ao semieixo positivo Oy, pelo que as suas coordenadas são da forma $(0,y,0)$, $y \in \mathbb{R}^+$.
Por outro lado, o ponto B pertence ao plano BDH, logo

$$0-7y+56=0 \Leftrightarrow y=8$$

Assim, $B(0,8,0)$.
Determinemos, agora, a medida de comprimento da aresta do cubo.

$$d(A,B)=\sqrt{(4-0)^2+(5-8)^2+(0-0)^2}=\sqrt{16+9}=\sqrt{25}=5$$

A face $[ABCD]$ está contida no plano xOy e $\overline{CG}=5$, daí que $G(3,12,-5)$.
O centro da superfície esférica de diâmetro $[EC]$ coincide com o centro do cubo, que é o ponto médio de $[AG]$.
Seja M esse ponto.

$$M\left(\frac{4+3}{2},\frac{5+12}{2},\frac{0-5}{2}\right) \Leftrightarrow M\left(\frac{7}{2},\frac{17}{2},-\frac{5}{2}\right)$$

$$\text{raio}=\frac{d(E,C)}{2}=\frac{d(A,G)}{2}=\frac{\sqrt{(4-3)^2+(5-12)^2+(0+5)^2}}{2}=\frac{\sqrt{1+49+25}}{2}=\frac{\sqrt{75}}{2}$$

Logo, $r^2=\frac{75}{4}$.
Assim, a equação da superfície esférica de diâmetro $[EC]$ é:

$$\left(x-\frac{7}{2}\right)^2+\left(y-\frac{17}{2}\right)^2+\left(z+\frac{5}{2}\right)^2=\frac{75}{4}$$

Resposta: **(C)**

1.2. Determinemos uma equação vetorial da reta EH.
O ponto A tem coordenadas $(4,5,0)$, pelo que, o ponto E tem coordenadas $(4,5,-5)$.
Por outro lado, $H=M+\overrightarrow{BM}$, sendo M o centro do cubo.
Ora, $\overrightarrow{BM}=M-B=\left(\frac{7}{2},\frac{17}{2},-\frac{5}{2}\right)-(0,8,0)=\left(\frac{7}{2},\frac{1}{2},-\frac{5}{2}\right)$.
Assim, $H=\left(\frac{7}{2},\frac{17}{2},-\frac{5}{2}\right)+\left(\frac{7}{2},\frac{1}{2},-\frac{5}{2}\right)=(7,9,-5)$.

Determinemos as coordenadas de $\overrightarrow{EH}$.

$\overrightarrow{EH} = H - E = (7,9,-5) - (4,5,-5) = (3,4,0)$

Portanto, $EH:\ (x,y,z) = (4,5,-5) + \lambda(3,4,0),\ \lambda \in \mathbb{R}$.

O ponto P é a interseção da reta EH com o plano de equação $y = -3$, pelo que:

$$\begin{cases} x = 4+3\lambda \\ y = 5+4\lambda \\ z = -5 \\ y = -3 \end{cases} \Leftrightarrow \begin{cases} x = 4+3\lambda \\ -3 = 5+4\lambda \\ z = -5 \\ y = -3 \end{cases} \Leftrightarrow \begin{cases} x = -2 \\ \lambda = -2 \\ z = -5 \\ y = -3 \end{cases}$$

Assim, $P(-2,-3,-5)$.

Determinemos a amplitudo do ângulo OPG.

$\overrightarrow{PO} = O - P = (2,3,5)$

$\overrightarrow{PG} = G - P = (3,12,-5) - (-2,-3,-5) = (5,15,0)$

$\overrightarrow{PO} \cdot \overrightarrow{PG} = (2,3,5)\cdot(5,15,0) = 2\times 5 + 3\times 15 + 5\times 0 = 10 + 45 = 55$

Resulta, assim, que:

$$\begin{aligned} & \overrightarrow{PO} \cdot \overrightarrow{PG} = \|\overrightarrow{PO}\| \times \|\overrightarrow{PG}\| \times \cos(O\hat{P}G) \\ \Leftrightarrow\ & 55 = \sqrt{2^2+3^2+5^2} \times \sqrt{5^2+15^2+0^2} \times \cos(O\hat{P}G) \\ \Leftrightarrow\ & 55 = \sqrt{4+9+25} \times \sqrt{25+225} \times \cos(O\hat{P}G) \\ \Leftrightarrow\ & 55 = \sqrt{38} \times \sqrt{250} \times \cos(O\hat{P}G) \\ \Leftrightarrow\ & 55 = \sqrt{9500} \times \cos(O\hat{P}G) \\ \Leftrightarrow\ & \cos(O\hat{P}G) = \frac{55}{\sqrt{9500}} \end{aligned}$$

Logo, $O\hat{P}G = \cos^{-1}\left(\frac{55}{\sqrt{9500}}\right) \approx 56°$.

2.

Sendo o polígono regular de 24 lados, tem-se que $A\hat{O}B = \frac{2\pi}{24} = \frac{\pi}{12}$.

Assim,

$$\begin{aligned} & \overrightarrow{OA} \cdot \overrightarrow{OB} = \|\overrightarrow{OA}\| \times \|\overrightarrow{OB}\| \times \cos(A\hat{O}B) \\ \Leftrightarrow\ & \frac{2}{\sqrt{6}-\sqrt{2}} = r \times r \times \cos\left(\frac{\pi}{12}\right), \text{ pois } \|\overrightarrow{OA}\| = \|\overrightarrow{OB}\| = \text{raio} = r \\ \Leftrightarrow\ & \frac{2}{\sqrt{6}-\sqrt{2}} = r^2 \times \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) \\ \Leftrightarrow\ & \frac{2}{\sqrt{6}-\sqrt{2}} = r^2 \times \left[\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)\right] \end{aligned}$$

$$\Leftrightarrow \frac{2}{\sqrt{6}-\sqrt{2}} = r^2 \times \left[\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}\right]$$
$$\Leftrightarrow \frac{2}{\sqrt{6}-\sqrt{2}} = r^2 \times \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$$
$$\Leftrightarrow \frac{2}{\sqrt{6}-\sqrt{2}} \times \frac{4}{\sqrt{6}+\sqrt{2}} = r^2$$
$$\Leftrightarrow \frac{8}{(\sqrt{6})^2-(\sqrt{2})^2} = r^2$$
$$\Leftrightarrow \frac{8}{6-2} = r^2$$
$$\Leftrightarrow r^2 = 2$$
$$\Leftrightarrow r = -\sqrt{2} \vee r = \sqrt{2}, \text{ como } r > 0,\ r = \sqrt{2}$$

Portanto, o perímetro desta circunferência é igual a $2\pi \times \sqrt{2} = 2\sqrt{2}\pi$.

Resposta: **(B)**

3.

- Números pares de um algarimo: 2, 4, 6 e 8.
  São 4 no total.
- Números pares de dois algarimos diferentes:

$$9 \times 1 + 8 \times 4 = 41$$

- Números pares de três algarismos diferentes:

$$^9A_2 + 8 \times 8 \times 4 = 328$$

- Números pares de quatro algarismos diferentes e inferiores a 4000:

$$2 \times {}^8A_2 \times 5 + 1 \times {}^8A_2 \times 4 = 784$$

Assim, $4 + 41 + 328 + 784 = 1157$.
Resposta: **(B)**

4.

Seja n o número de medalhas de prata.

O número de casos possíveis é igual a $(n+19)!$.

O número de casos favoráveis é igual a $^5A_2 \times (n+17)!$

Tem-se assim, pela regra de Laplace, que:

$$
\begin{aligned}
& \frac{{}^{5}A_{2} \times (n+17)!}{(n+19)!} = \frac{5}{189} \\
\Leftrightarrow\ & \frac{20(n+17)!}{(n+19)(n+18)(n+17)!} = \frac{5}{189} \\
\Leftrightarrow\ & \frac{20}{(n+19)(n+18)} = \frac{5}{189} \\
\Leftrightarrow\ & (n+19)(n+18) = \frac{189 \times 20}{5} \\
\Leftrightarrow\ & n^2 + 19n + 18n + 342 = 756 \\
\Leftrightarrow\ & n^2 + 37n - 414 = 0 \\
\Leftrightarrow\ & n = \frac{-37 \pm \sqrt{1369 - 4(-414)}}{2} \\
\Leftrightarrow\ & n = \frac{-37-55}{2} \vee n = \frac{-37+55}{2} \\
\Leftrightarrow\ & n = -46 \vee n = 9
\end{aligned}
$$

Logo, o número de medalhas de prata é 9.

5.

Sejam A e B acontecimentos tais que:

A: "o funcionário está vacinado contra a covid-19"

B: "o funcionário tem menos de 40 anos de idade "

Assim, tem-se que:

$$P(A) = 0,88;\ P(B|A) = \frac{2}{11} \text{ e } P(B|\overline{A}) = \frac{2}{3}$$

Pretende-se determinar $P(A|\overline{B})$.

Ora, $P(B|A) = \dfrac{P(B \cap A)}{P(A)} \Leftrightarrow P(B \cap A) = P(B|A) \times P(A)$.

Logo, $P(B \cap A) = \dfrac{2}{11} \times 0,88 = 0,16$.

Analogamente, $P(B|\overline{A}) = \dfrac{P(B \cap \overline{A})}{P(\overline{A})} \Leftrightarrow P(B \cap \overline{A}) = P(B|\overline{A}) \times P(\overline{A})$.

Logo, $P(B \cap \overline{A}) = \dfrac{2}{3} \times (1 - 0,88) = 0,08$.

Recorrendo a uma tabela:

	A	$\overline{A}$	Total
B	$0,16$	$0,08$	$0,24$
$\overline{B}$	$0,72$	$0,04$	$0,76$
Total	$0,88$	$0,12$	1

Assim, resulta que $P(A|\overline{B}) = \dfrac{P(A \cap \overline{B})}{P(\overline{B})} = \dfrac{0,72}{0,76} = \dfrac{18}{19}$.

6.

Sendo (u_n) a progressão aritmética, tal que:

- $u_5 = u_3 + 2r \Leftrightarrow 2a + 4 = a + 1 + 2r \Leftrightarrow a = 2r - 3$
- $S_{13} = 208 \Leftrightarrow \dfrac{u_1 + u_{13}}{2} \times 13 = 208 \Leftrightarrow \dfrac{u_3 - 2r + u_3 + 10r}{2} \times 13 = 208$

Assim, temos que:

$$\frac{u_3 - 2r + u_3 + 10r}{2} \times 13 = 208 \Leftrightarrow \frac{a + 1 - 2r + a + 1 + 10r}{2} \times 13 = 208$$

Sendo $a = 2r - 3$ temos:

$$\begin{aligned}
& \frac{a + 1 - 2r + a + 1 + 10r}{2} \times 13 = 208 \\
\Leftrightarrow\ & \frac{2r - 3 + 1 - 2r + 2r - 3 + 1 + 10r}{2} \times 13 = 208 \\
\Leftrightarrow\ & \frac{12r - 4}{2} \times 13 = 208 \\
\Leftrightarrow\ & 12r - 4 = \frac{208 \times 2}{13} \\
\Leftrightarrow\ & 12r - 4 = 32 \\
\Leftrightarrow\ & 12r = 36 \\
\Leftrightarrow\ & r = 3
\end{aligned}$$

Resposta: **(B)**

7.

Ora, $u_n = \dfrac{(-1)^{n+1} \times 6n}{1+2n} \Leftrightarrow u_n = \begin{cases} \dfrac{6n}{1+2n} & \text{se } n \text{ é ímpar} \\ \dfrac{-6n}{1+2n} & \text{se } n \text{ é par} \end{cases}$

- Para n ímpar:

$$\begin{array}{ll|l} 6n & & 2n+1 \\ \hline 6n & -3 & 3 \\ & -3 & \end{array}$$

Assim, $\dfrac{6n}{2n+1} = 3 - \dfrac{3}{2n+1}$.

$$\begin{aligned} \forall n \in \mathbb{N} \wedge n \text{ ímpar} : 0 < \frac{3}{2n+1} \leq 1 \\ \Leftrightarrow \quad -1 \leq -\frac{3}{2n+1} < 0 \\ \Leftrightarrow \quad 2 \leq 3 - \frac{3}{2n+1} < 3 \end{aligned}$$

- Para n par:

$$\begin{array}{ll|l} -6n & & 2n+1 \\ \hline 6n & +3 & -3 \\ & 3 & \end{array}$$

Assim, $\dfrac{-6n}{2n+1} = -3 + \dfrac{3}{2n+1}$.

$$\begin{aligned} \forall n \in \mathbb{N} \wedge n \text{ par} : 0 < \frac{3}{2n+1} \leq \frac{3}{5} \\ \Leftrightarrow \quad -3 < -3 + \frac{3}{2n+1} \leq -\frac{12}{5} \end{aligned}$$

Portanto, $\forall n \in \mathbb{N},\ -3 < u_n < 3$, logo (u_n) é limitada, pois é minorada e majorada.

Conjunto dos minorantes = $]-\infty, -3]$

Conjunto dos majorantes = $[3, +\infty[$

8.

$$z_2 = z_1 + 1 = e^{i\theta} + 1 = \cos\theta + i\sin\theta + 1 = (\cos\theta + 1) + i\sin\theta$$

$$\begin{aligned}
|z_2| &= \sqrt{(\cos\theta+1)^2 + \sin^2\theta} = \sqrt{\cos^2\theta + 2\cos\theta + 1 + \sin^2\theta} \\
&= \sqrt{2\cos\theta + 2}, \text{ pois } \cos^2\theta + \sin^2\theta = 1 \\
&= \sqrt{2(1+\cos\theta)} \\
&= \sqrt{2\left(1 + \cos\left(\frac{\theta}{2} + \frac{\theta}{2}\right)\right)} \\
&= \sqrt{2\left(1 + \cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\right)} \\
&= \sqrt{2\left(1 + \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)} \\
&= \sqrt{2\left(1 - \sin^2\left(\frac{\theta}{2}\right) + \cos^2\left(\frac{\theta}{2}\right)\right)} \\
&= \sqrt{2\left(\cos^2\left(\frac{\theta}{2}\right) + \cos^2\left(\frac{\theta}{2}\right)\right)} \\
&= \sqrt{2\left(2\cos^2\left(\frac{\theta}{2}\right)\right)} \\
&= \sqrt{4\cos^2\left(\frac{\theta}{2}\right)} \\
&= 2\sqrt{\cos^2\left(\frac{\theta}{2}\right)} = 2\left|\cos\left(\frac{\theta}{2}\right)\right|
\end{aligned}$$

Como $\theta \in]0,\pi] \Rightarrow \frac{\theta}{2} \in \left]0, \frac{\pi}{2}\right]$, pelo que, $\cos\left(\frac{\theta}{2}\right) \geq 0$, assim: $2\left|\cos\left(\frac{\theta}{2}\right)\right| = 2\cos\left(\frac{\theta}{2}\right)$.

$$\begin{aligned}
\tan[\arg(z_2)] &= \frac{\sin\theta}{1+\cos\theta} \\
&= \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)} \\
&= \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} \\
&= \tan\left(\frac{\theta}{2}\right)
\end{aligned}$$

Portanto, $\arg(z_2) = \dfrac{\theta}{2}$, donde resulta que: $z_2 = z_1 + 1 \Leftrightarrow z_2 = 2\cos\left(\dfrac{\theta}{2}\right)e^{i\frac{\theta}{2}}$.

Resposta: **(C)**

9.

$$
\begin{aligned}
& z_1 = \frac{1+i}{i} + \frac{i}{1+i} + i^{31} \\
\Leftrightarrow\ & z_1 = \frac{i(1+i)}{i^2} + \frac{i(1-i)}{(1+i)(1-i)} + i^3 \\
\Leftrightarrow\ & z_1 = \frac{i+i^2}{-1} + \frac{i-i^2}{1^2-i^2} - i \\
\Leftrightarrow\ & z_1 = \frac{i-1}{-1} + \frac{i+1}{2} - i \\
\Leftrightarrow\ & z_1 = 1 - i + \frac{1}{2}i + \frac{1}{2} - i \\
\Leftrightarrow\ & z_1 = \frac{3}{2} - \frac{3}{2}i
\end{aligned}
$$

Então, $|z_1| = \sqrt{\left(\dfrac{3}{2}\right)^2 + \left(-\dfrac{3}{2}\right)^2} = \sqrt{\dfrac{9}{4}+\dfrac{9}{4}} = \sqrt{\dfrac{18}{4}} = \dfrac{3\sqrt{2}}{2}$.

$\tan\theta = \dfrac{-\frac{3}{2}}{\frac{3}{2}} = -1 \wedge \theta \in 4^{\text{o}}$ Quadrante $\Rightarrow \theta = -\dfrac{\pi}{4}$

Logo, $z_1 = \dfrac{3\sqrt{2}}{2}e^{i\left(-\frac{\pi}{4}\right)}$.

$$
\begin{aligned}
& z_2 = -\sin\frac{\pi}{7} + i\cos\left(\frac{\pi}{7}\right) \\
\Leftrightarrow\ & z_2 = \cos\left(\frac{\pi}{2}+\frac{\pi}{7}\right) + i\sin\left(\frac{\pi}{2}+\frac{\pi}{7}\right) \\
\Leftrightarrow\ & z_2 = \cos\left(\frac{9\pi}{14}\right) + i\sin\left(\frac{9\pi}{14}\right) \\
\Leftrightarrow\ & z_2 = e^{i\frac{9\pi}{14}}
\end{aligned}
$$

Assim:

$$
\left(\frac{z_1}{z_2}\right)^n = \left(\frac{\frac{3\sqrt{2}}{2}e^{i\left(-\frac{\pi}{4}\right)}}{e^{i\frac{9\pi}{14}}}\right)^n = \left(\frac{3\sqrt{2}}{2}\right)^n e^{i\left(-\frac{\pi}{4}-\frac{9}{14}\right)n} = \left(\frac{3\sqrt{2}}{2}\right)^n e^{i\left(-\frac{25\pi}{28}\right)n}
$$

Ora, $\left(\dfrac{z_1}{z_2}\right)^n \in \mathbb{R}^+$ quando $\arg\left(\dfrac{z_1}{z_2}\right)^n = 2k\pi,\ k \in \mathbb{Z}$, pelo que:

$$-\frac{25}{28}\pi n = 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow n = -\frac{56}{25}k,\ k \in \mathbb{Z}$$

Se $k = -25 \Rightarrow n = 56$.

10.

10.1. f é contínua em $x = 2$ se e só se:

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = f(2)$$

$\lim\limits_{x \to 2^-} f(x) = f(2) = 2 \times 2 - 2^2 \times e^{2-2} = 4 - 4 = 0$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} \dfrac{\ln(x-1)}{x^2 + x - 6} \overset{(1)}{=} \lim\limits_{y \to 0^+} \dfrac{y}{(e^y+1)^2 + e^y + 1 - 6} = \lim\limits_{y \to 0^+} \dfrac{y}{e^{2y} + 2e^y + 1 + e^y - 5} =$
$= \lim\limits_{y \to 0^+} \dfrac{y}{e^{2y} + 3e^y - 4}$

(1) $y = \ln(x-1)$, logo, $\ln(e^y) = \ln(x-1) \Leftrightarrow e^y = x - 1 \Leftrightarrow e^y + 1 = x$; $x \to 2^+ \Rightarrow y \to 0^+$

Cálculos auxiliares:
$e^{2y} + 3e^y - 4 = 0 \Leftrightarrow e^y = \dfrac{-3 \pm \sqrt{9 - 4(-4)}}{2} \Leftrightarrow e^y = 1 \vee e^y = -4$

Assim,

$$\lim_{y \to 0^+} \frac{y}{e^{2y} + 3e^y - 4} = \lim_{y \to 0^+} \frac{y}{(e^y - 1)(e^y + 4)}$$
$$= \lim_{y \to 0^+} \frac{y}{e^y - 1} \times \lim_{y \to 0^+} \frac{y}{e^y + 4}$$
$$= \frac{1}{\lim\limits_{y \to 0^+} \dfrac{e^y - 1}{y}} \times \frac{1}{e^0 + 4}$$
$$= \frac{1}{1} \times \frac{1}{5} = \frac{1}{5}$$

Como $\lim\limits_{x \to 2^+} f(x) \neq \lim\limits_{x \to 2^-} f(x)$, então f não é contínua em $x = 2$.

10.2.

$m = \lim\limits_{x \to -\infty} \dfrac{f(x)}{x} = \lim\limits_{x \to -\infty} \dfrac{2x - x^2 e^{x-2}}{x} = \lim\limits_{x \to -\infty} \dfrac{2x}{x} - \lim\limits_{x \to -\infty} \dfrac{x^2 e^{x-2}}{x} = 2 - \lim\limits_{x \to -\infty} x e^{x-2} \overset{(2)}{=}$

$= 2 - \lim\limits_{y \to +\infty} (-y e^{-y-2}) = 2 + \lim\limits_{y \to +\infty} \dfrac{y}{e^{y+2}} = 2 + \lim\limits_{y \to +\infty} \dfrac{y}{e^y \times e^2} = 2 + \dfrac{1}{e^2} \times \dfrac{1}{\lim\limits_{y \to +\infty} \dfrac{e^y}{y}} =$

$= 2 + \dfrac{1}{e^2} \times \dfrac{1}{+\infty} = 2 + \dfrac{1}{e^2} \times 0 = 2$

(2) $\quad y=-x \Leftrightarrow x=-y\ ; \quad x \to -\infty \Rightarrow y \to +\infty$

$$b=\lim_{x\to-\infty}(f(x)-mx)=\lim_{x\to-\infty}(2x-x^2e^{x-2}-2x)=\lim_{x\to-\infty}(-x^2e^{x-2})\overset{(3)}{=}-\lim_{y\to+\infty}((-y)^2e^{-y-2})=$$

$$=-\lim_{y\to+\infty}\frac{y^2}{e^{y+2}}=-\lim_{y\to+\infty}\frac{y^2}{e^y\times e^2}=-\frac{1}{e^2}\times\frac{1}{\lim\limits_{y\to+\infty}\frac{e^y}{y^2}}=-\frac{1}{e^2}\times\frac{1}{+\infty}=-\frac{1}{e^2}\times 0=0$$

(3) $\quad y=-x \Leftrightarrow x=-y\ ; \quad x \to -\infty \Rightarrow y \to +\infty$

A reta de equação $y=2x$ é assíntota oblíqua ao gráfico de f, em $]-\infty,2[$.

11.

A reta de equação $y=-2x+3$ é assíntota ao oblíqua ao gráfico de f em $+\infty$, pelo que:

$$\lim_{x\to+\infty}\frac{f(x)}{x}=-2 \text{ e } \lim_{x\to+\infty}(f(x)+2x)=3$$

Seja $y=mx+b$ a equação da assíntota ao gráfico de f em $-\infty$.

Ora,

$$m=\lim_{x\to-\infty}\frac{f(x)}{x}\overset{(4)}{=}\lim_{y\to+\infty}\frac{f(-y)}{-y}\overset{(5)}{=}\lim_{y\to+\infty}\frac{-f(y)}{-y}=\lim_{y\to+\infty}\frac{f(y)}{y}=-2$$

$$b=\lim_{x\to-\infty}(f(x)-mx)=\lim_{x\to-\infty}(f(x)+2x)\overset{(4)}{=}\lim_{y\to+\infty}[f(-y)+2(-y)]\overset{(5)}{=}$$
$$=\lim_{y\to+\infty}(-f(y)-2y)=-\lim_{y\to+\infty}(f(y)+2y)=-3$$

(4) $\quad y=-x \Leftrightarrow x=-y\ ; \quad x \to -\infty \Rightarrow y \to +\infty$

(5) $\quad f$ é uma função ímpar se e só se $\forall x\in D_f, -x\in D_f, f(-x)=-f(x)$

A reta de equação $y=-2x-3$ é assíntota ao gráfico de f em $-\infty$.

12.

$$\begin{aligned}h''(x) &= [1-(\ln(x)-1)^2]' \\ &= (1)' - [(\ln(x)-1)^2]' \\ &= 0 - 2(\ln(x)-1)(\ln(x)-1)' \\ &= -2(\ln(x)-1)\frac{1}{x} \\ &= -\frac{2}{x}(\ln(x)-1) \\ &= \frac{-2\ln(x)+2}{x}\end{aligned}$$

Zeros de h'':

$$\begin{aligned}h''(x)=0 &\Leftrightarrow \frac{-2\ln(x)+2}{x}=0 \Leftrightarrow -2\ln x+2=0 \wedge x\neq 0 \Leftrightarrow \ln x = 1 \wedge x\neq 0 \\ &\Leftrightarrow x=e \wedge x\neq 0 \Leftrightarrow x=e\end{aligned}$$

x	0		e	$+\infty$
Sinal de h''		$+$	0	$-$
Sentido da convidade do gráfico de h		$\cup$	**P.I.**	$\cap$

O gráfico de h tem concavidade voltada para cima em $]0,e]$ e voltada para baixo em $[e,+\infty[$.

O gráfico de h tem só um ponto de inflexão de abcissa igual a e.

13.

$$D=\left\{x\in\mathbb{R}:\ \frac{e^x-2}{1-3e^x}>0\right\}$$

- $e^x-2=0 \Leftrightarrow e^x=2 \Leftrightarrow e^x=e^{\ln 2} \Leftrightarrow x=\ln 2$
- $1-3e^x=0 \Leftrightarrow e^x=\dfrac{1}{3} \Leftrightarrow e^x=e^{\ln\left(\frac{1}{3}\right)} \Leftrightarrow x=\ln\left(\dfrac{1}{3}\right)$

x	$-\infty$	$\ln\left(\frac{1}{3}\right)$		$\ln 2$	$+\infty$
e^x-2	$-$	$-$	$-$	0	$+$
$1-3e^x$	$+$	0	$-$	$-$	$-$
$\frac{e^x-2}{1-3e^x}$	$-$	n.d.	$+$	0	$-$

Logo, $D = \left]\ln\left(\frac{1}{3}\right), \ln 2\right[$.

Assim, temos:

$$\ln\left(\frac{e^x-2}{1-3e^x}\right) \leq 0 \wedge x \in D \Leftrightarrow \ln\left(\frac{e^x-2}{1-3e^x}\right) \leq \ln e^0 \wedge x \in D$$

$$\Leftrightarrow \frac{e^x-2}{1-3e^x} \leq 1 \wedge x \in D \Leftrightarrow e^x - 2 \geq 1 - 3e^x \wedge x \in D, \text{ pois } \forall x \in D,\ 1-3e^x < 0$$

$$\Leftrightarrow 4e^x \geq 3 \wedge x \in D \Leftrightarrow \quad e^x \geq \frac{3}{4} \wedge x \in D$$

$$\Leftrightarrow e^x \geq e^{\ln\left(\frac{3}{4}\right)} \wedge x \in D \Leftrightarrow x \geq \ln\left(\frac{3}{4}\right) \wedge x \in D$$

$$\Leftrightarrow \ln\left(\frac{3}{4}\right) \leq x < \ln 2$$

Conjunto solução = $\left[\ln\left(\frac{3}{4}\right), \ln 2\right[$

14.

14.1. α é a inclinação da reta OB, portanto, $\tan\alpha = -\frac{\sqrt{3}}{3} \wedge \theta \in [0,\pi[\Rightarrow \alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

O ponto A tem coordendadas $(2,0)$ e o ponto B tem coordendadas $\left(2\cos\left(\frac{5\pi}{6}\right), 2\sin\left(\frac{5\pi}{6}\right)\right)$, isto é, $B(-\sqrt{3},1)$.

$$d(A,B) = \sqrt{(2+\sqrt{3})^2+(0-1)^2} = \sqrt{4+4\sqrt{3}+3+1} = \sqrt{4\sqrt{3}+8} = \sqrt{4(\sqrt{3}+2)} = 2\left(\sqrt{\sqrt{3}+2}\right)$$

Perímetro $\Delta[AOB] = d(O,A)+d(O,B)+d(A,B) = 2+2+2\left(\sqrt{\sqrt{3}+2}\right) = 4+2\left(\sqrt{\sqrt{3}+2}\right)$

Resposta: **(B)**

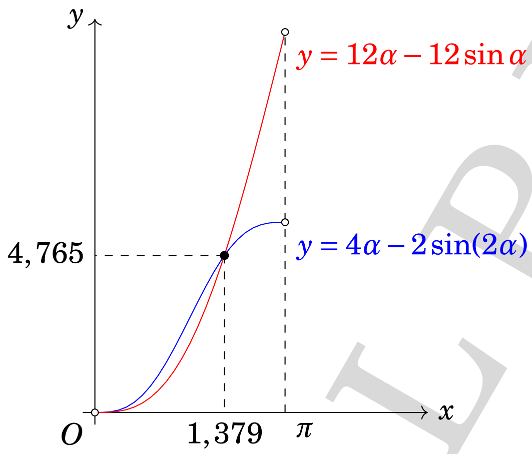
14.2.

$$A(\alpha) = A_{\text{setor circular}AOB} - A_{\Delta[AOB]} = \frac{\alpha \times 2^2}{2} - \frac{2 \times 2\sin\alpha}{2} = 2\alpha - 2\sin\alpha$$

Equação que traduz o problema:

$$A(2\alpha) = 6A(\alpha)$$

Portanto, $2(2\alpha) - 2\sin(2\alpha) = 6(2\alpha - 2\sin\alpha) \Leftrightarrow 4\alpha - 2\sin(2\alpha) = 12\alpha - 12\sin\alpha$



Portanto, $\alpha \approx 1,38$ rad.

15.

$$\begin{aligned}
f'(x) &= (2\cos x(1+\sin x))' \\
&= (2\cos x)'(1+\sin x) + (2\cos x)(1+\sin x)' \\
&= (-2\sin x)(1+\sin x) + (2\cos x)(\cos x) \\
&= -2\sin x - 2\sin^2 x + 2\cos^2 x \\
&= -2\sin x - 2\sin^2 x + 2(1-\sin^2 x) \\
&= -2\sin x - 2\sin^2 x + 2 - 2\sin^2 x \\
&= -4\sin^2 x - 2\sin x + 2
\end{aligned}$$

Zeros de f':

$$\begin{aligned}
& f'(x) = 0 \\
\Leftrightarrow\ & -4\sin^2 x - 2\sin x + 2 = 0 \\
\Leftrightarrow\ & 2\sin^2 x + \sin x - 1 = 0 \\
\Leftrightarrow\ & \sin x = \frac{-1 \pm \sqrt{1-4(2)(-1)}}{4} \\
\Leftrightarrow\ & \sin x = \frac{-1-3}{4} \vee \sin x = \frac{-1+3}{4} \\
\Leftrightarrow\ & \sin x = -1 \vee \sin x = \frac{1}{2}
\end{aligned}$$

Como $x \in]0, \pi[\Rightarrow x = \frac{\pi}{6} \vee x = \frac{5\pi}{6}$

x	0		$\frac{\pi}{6}$		$\frac{5\pi}{6}$		π
Sinal de f'		$+$	0	$-$	0	$+$	
Variação de f		$\nearrow$	máx	$\searrow$	min	$\nearrow$	

Ordenada de $A = f\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right)\left(1+\sin\left(\frac{\pi}{6}\right)\right) = 2\left(\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right) = \frac{3\sqrt{3}}{2}$.

Logo, $A\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right)$ e portanto $C\left(0, \frac{3\sqrt{3}}{2}\right)$.

Ordenada de $B = f\left(\frac{5\pi}{6}\right) = 2\cos\left(\frac{5\pi}{6}\right)\left(1+\sin\left(\frac{5\pi}{6}\right)\right) = 2\left(-\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right) = -\frac{3\sqrt{3}}{2}$.

Logo, $B\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right)$ e portanto $D\left(0, -\frac{3\sqrt{3}}{2}\right)$.

Assim, $A_{[ABCD]} = \frac{\overline{AC}+\overline{BD}}{2} \times \overline{CD} = \frac{\frac{\pi}{6}+\frac{5\pi}{6}}{2} \times 3\sqrt{3} = \frac{\pi}{2} \times 3\sqrt{3} = \frac{3\sqrt{3}\pi}{2}$.

FIM