11º ANO | PROPOSTA RESOLUÇÃO TESTE 2 | 2023

António Leite

1.

1.1. Temos que $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DE} = \overset{\frown}{EA} = \dfrac{360°}{5} = 72° \left(\dfrac{2\pi}{5} \text{ em radianos}\right)$

1.1.1. $-216° = 72° \times (-3)$

Resposta: $\dot{O}B$

1.1.2. $\dfrac{24\pi}{5} = \dfrac{20\pi}{5} + \dfrac{4\pi}{5} = 4\pi + \dfrac{2\pi}{5} \times 2$

Resposta: $\dot{O}B$

1.2. $\text{Área}_{[ABCO]} = 2 \times \text{Área}_{[AOB]}$ (1)

Ora, $\overset{\frown}{AB} = 72°$, portanto $A\hat{O}B = 72°$.

Sendo M o ponto médio de $[AB]$, então $A\hat{O}M = \dfrac{72°}{2} = 36°$.

Portanto, $\cos 36° = \dfrac{\overline{OM}}{\overline{AO}} \Leftrightarrow \cos 36° = \dfrac{\overline{OM}}{2} \Leftrightarrow \overline{OM} = 2 \times \cos 36°$.

Por outro lado, $\sin 36° = \dfrac{\overline{AM}}{\overline{AO}} \Leftrightarrow \sin 36° = \dfrac{\overline{AM}}{2} \Leftrightarrow \overline{AM} = 2 \times \sin 36°$.

Voltando a (1)

$$\begin{aligned}
2 \times \text{Área}_{[AOB]} &= 2 \times \frac{\overline{AB} \times \overline{OM}}{2} \\
&= \overline{AB} \times \overline{OM} \\
&= 2 \times \overline{AM} \times \overline{OM} \\
&= 2 \times 2\sin 36° \times 2\cos 36° \\
&= 8 \sin 36° \cos 36° \approx 3{,}8
\end{aligned}$$

2.

Temos que

- $\cos(-\pi-\alpha)=\dfrac{1}{3} \Leftrightarrow -\cos\alpha=\dfrac{1}{3} \Leftrightarrow \cos\alpha=-\dfrac{1}{3}$

  Como $\cos\alpha<0 \wedge \alpha\in]0,\pi[\Rightarrow \alpha\in\left]\dfrac{\pi}{2},\pi\right[$.

- $\cos\left(\dfrac{3\pi}{2}-\alpha\right)-2\tan(-\pi+\alpha)=-\sin\alpha-2\tan\alpha \quad (2)$

Recorrendo às fórmulas trigonométricas, vem que:

- $\cos^2\alpha+\sin^2\alpha=1$

  $\left(-\dfrac{1}{3}\right)^2+\sin^2\alpha=1 \Leftrightarrow \dfrac{1}{9}+\sin^2\alpha=1 \Leftrightarrow \sin^2\alpha=\dfrac{8}{9} \Leftrightarrow \sin\alpha=\pm\dfrac{2\sqrt{2}}{3}$

Como $\alpha\in\left]\dfrac{\pi}{2},\pi\right[$, então $\sin\alpha>0$, pelo que $\sin\alpha=\dfrac{2\sqrt{2}}{3}$.

- $\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}$

  $\tan\alpha=\dfrac{\frac{2\sqrt{2}}{3}}{-\frac{1}{3}} \Leftrightarrow \tan\alpha=-2\sqrt{2}$

Voltando a (2) temos que:

$$-\sin\alpha-2\tan\alpha=-\frac{2\sqrt{2}}{3}-2(-2\sqrt{2})=-\frac{2\sqrt{2}}{3}+4\sqrt{2}=-\frac{2\sqrt{2}}{3}+\frac{12\sqrt{2}}{3}=\frac{10}{3}\sqrt{2}$$

3.

Temos que, $\theta\in\left]-\dfrac{3\pi}{2},-\pi\right[$, ou seja $\theta\in 2^{\text{o}}$ quadrante.

(A) $\sin\left(\dfrac{\pi}{2}+\theta\right)=\cos\theta<0$, pois $\theta\in 2^{\text{o}}$Q.

(B) $\tan(-\pi+\theta)=\tan\theta<0$, pois $\theta\in 2^{\text{o}}$Q

(C) $\cos\left(-\dfrac{\pi}{2}-\theta\right)=-\sin\theta<0$, pois $\theta\in 2^{\text{o}}$Q

(D) $\cos(-\theta-\pi)=-\cos\theta>0$, pois $\theta\in 2^{\text{o}}$Q

Resposta: **(D)**

4.

4.1.

$$\sqrt{2}\cos\left(2x-\frac{\pi}{3}\right)+1=0$$

$$\Leftrightarrow \cos\left(2x-\frac{\pi}{3}\right)=-\frac{1}{\sqrt{2}}$$

$$\Leftrightarrow \cos\left(2x-\frac{\pi}{3}\right)=-\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow 2x-\frac{\pi}{3}=\frac{3\pi}{4}+2k\pi \vee 2x-\frac{\pi}{3}=\frac{5\pi}{4}+2k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow 2x=\frac{3\pi}{4}+\frac{\pi}{3}+2k\pi \vee 2x=\frac{5\pi}{4}+\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow 2x=\frac{9\pi}{12}+\frac{4\pi}{12}+2k\pi \vee 2x=\frac{15\pi}{12}+\frac{4\pi}{12}+2k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow 2x=\frac{13\pi}{12}+2k\pi \vee 2x=\frac{19\pi}{12}+2k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow x=\frac{13\pi}{24}+k\pi \vee x=\frac{19\pi}{24}+k\pi,\ k\in\mathbb{Z}$$

4.2.

$$4\sin\left(\frac{\pi}{3}-3x\right)=\sqrt{12}$$

$$\Leftrightarrow \sin\left(\frac{\pi}{3}-3x\right)=\frac{2\sqrt{3}}{4}$$

$$\Leftrightarrow \sin\left(\frac{\pi}{3}-3x\right)=\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \frac{\pi}{3}-3x=\frac{\pi}{3}+2k\pi \vee \frac{\pi}{3}-3x=\frac{2\pi}{3}+2k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow -3x=\frac{\pi}{3}-\frac{\pi}{3}+2k\pi \vee -3x=\frac{2\pi}{3}-\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow -3x=2k\pi \vee -3x=\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow x=-\frac{2k\pi}{3} \vee x=-\frac{\pi}{9}-\frac{2k\pi}{3},\ k\in\mathbb{Z}$$

4.3.

$$\tan\left(2x+\frac{\pi}{5}\right)=\tan(\pi-x)$$

$$\Leftrightarrow\ 2x+\frac{\pi}{5}=\pi-x+k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow\ 2x+x=\pi-\frac{\pi}{5}+k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow\ 3x=\frac{5\pi}{5}-\frac{\pi}{5}+k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow\ 3x=\frac{4\pi}{5}+k\pi,\ k\in\mathbb{Z}$$

$$\Leftrightarrow\ x=\frac{4\pi}{15}+\frac{k\pi}{3},\ k\in\mathbb{Z}$$

5.

Temos que:

$$\cos\left(\frac{23\pi}{6}\right)+\sin\left(-\frac{20\pi}{3}\right)+a=\tan\left(-\frac{7\pi}{6}\right)$$

$$\Leftrightarrow\ \cos\left(\frac{12\pi}{6}+\frac{11\pi}{6}\right)+\sin\left(-\frac{18\pi}{3}-\frac{2\pi}{3}\right)+a=\tan\left(-\frac{7\pi}{6}\right)$$

$$\Leftrightarrow\ \cos\left(\frac{11\pi}{6}\right)+\sin\left(-\frac{2\pi}{3}\right)+a=\tan\left(-\frac{7\pi}{6}\right)$$

$$\Leftrightarrow\ \cos\left(\frac{\pi}{6}\right)-\sin\left(\frac{\pi}{3}\right)+a=-\tan\left(\frac{\pi}{6}\right)$$

$$\Leftrightarrow\ \frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}+a=-\frac{\sqrt{3}}{3}$$

$$\Leftrightarrow\ a=-\frac{\sqrt{3}}{3}$$

E, ainda:

$$\cos\left(-\frac{13\pi}{4}\right)-2\sin\left(-\frac{19\pi}{6}\right)+b=\tan\left(-\frac{9\pi}{4}\right)$$

$$\Leftrightarrow\ \cos\left(-\frac{8\pi}{4}-\frac{5\pi}{4}\right)-2\sin\left(-\frac{12\pi}{6}-\frac{7\pi}{6}\right)+b=\tan\left(-\frac{8\pi}{4}-\frac{\pi}{4}\right)$$

$$\Leftrightarrow\ \cos\left(-\frac{5\pi}{4}\right)-2\sin\left(-\frac{7\pi}{6}\right)+b=\tan\left(-\frac{\pi}{4}\right)$$

$$\Leftrightarrow\ -\cos\left(\frac{\pi}{4}\right)-2\sin\left(\frac{\pi}{6}\right)+b=-\tan\left(\frac{\pi}{4}\right)$$

$$\Leftrightarrow\ -\frac{\sqrt{2}}{2}-2\times\frac{1}{2}+b=-1$$
$$\Leftrightarrow\ -\frac{\sqrt{2}}{2}-1+b=-1$$
$$\Leftrightarrow\ b=\frac{\sqrt{2}}{2}$$

Logo, $a=-\dfrac{\sqrt{3}}{3}$ e $b=\frac{\sqrt{2}}{2}$, portanto:

$$\frac{a}{b}=\frac{\frac{-\sqrt{3}}{3}}{\frac{\sqrt{2}}{2}}=-\frac{\sqrt{3}}{3}\times\frac{2}{\sqrt{2}}=-\frac{2\sqrt{3}}{3\sqrt{2}}=-\frac{2\sqrt{3}\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}}=-\frac{2\sqrt{6}}{3\times 2}=-\frac{\sqrt{6}}{3}$$

6.

Ora, P tem ordenada $-\dfrac{15}{17}$, pelo que $\sin\alpha=-\dfrac{15}{17}$.

A abcissa do ponto P é igual a $\cos\alpha$.

Recorrendo à fórmula fundamental da trigonometria, vem que:

$$\cos^2\alpha+\sin^2\alpha=1$$
$$\cos^2\alpha+\left(-\frac{15}{17}\right)^2=1\Leftrightarrow\cos^2\alpha+\frac{225}{289}=1\Leftrightarrow\cos^2\alpha=\frac{289}{289}-\frac{225}{289}\Leftrightarrow\cos^2\alpha=\frac{64}{289}\Leftrightarrow$$
$$\Leftrightarrow\ \cos\alpha=\pm\sqrt{\frac{64}{289}}\Leftrightarrow\cos\alpha=\pm\frac{8}{17}$$

Como $\alpha\in 4^{\text{o}}$ Quadrante, $\cos\alpha>0$, pelo que a abcissa do ponto P é $\dfrac{8}{17}$.

A ordenada do ponto T é igual a $\tan\alpha$.

Assim, vem que:

$\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}$, ou seja, $\tan\alpha=\dfrac{-\frac{15}{17}}{\frac{8}{17}}\Leftrightarrow\tan\alpha=-\dfrac{15}{8}$

Portanto, a ordenada do ponto T é $-\dfrac{15}{8}$.

7.

$$
\begin{aligned}
& 2\cos^2(x) - 1 = 0 \Leftrightarrow \\
\Leftrightarrow\ & \cos^2(x) = \frac{1}{2} \Leftrightarrow \\
\Leftrightarrow\ & \cos x = -\sqrt{\frac{1}{2}} \vee \cos x = \sqrt{\frac{1}{2}} \Leftrightarrow \\
\Leftrightarrow\ & \cos x = -\frac{\sqrt{2}}{2} \vee \cos x = \frac{\sqrt{2}}{2} \Leftrightarrow \\
\Leftrightarrow\ & x = \frac{3\pi}{4} + 2k\pi \vee x = \frac{5\pi}{4} + 2k\pi \vee x = \frac{\pi}{4} + 2k\pi \vee x = \frac{7\pi}{4} + 2k\pi, k \in \mathbb{Z} \\
\Leftrightarrow\ & x = \frac{\pi}{4} + k\frac{\pi}{2}, k \in \mathbb{Z}
\end{aligned}
$$

Resposta: **(C)**

8.

8.1. Coordenadas dos vértices do trapézio $[OABC]$:

- $A(3\cos\alpha, 3\sin\alpha)$
- $B(3\cos\alpha, -3\sin\alpha)$
- $C(0, -3\sin\alpha)$
- $O(0,0)$

Por outro lado, temos que:

$$\text{Área}_{[OABC]} = \frac{\overline{AB} + \overline{OC}}{2} \times \overline{BC}$$

Ora,

- $\overline{AB} = 2 \times y_A = 2 \times 3\sin\alpha = 6\sin\alpha$
- $\overline{OC} = \frac{\overline{AB}}{2} = \frac{6\sin\alpha}{2} = 3\sin\alpha$
- $\overline{BC} = -x_A = -3\cos\alpha$

Portanto,

$$
\begin{aligned}
\text{Área}_{[OABC]} &= \frac{6\sin\alpha + 3\sin\alpha}{2} \times (-3\cos\alpha) \\
&= \frac{9\sin\alpha}{2} \times (-3\cos\alpha) \\
&= -\frac{27}{2}\sin\alpha\cos\alpha \qquad \text{(c.q.m)}
\end{aligned}
$$

8.2. Sendo $\alpha = \dfrac{5\pi}{6}$, vem que:

$$\begin{aligned}
\text{Área}_{[OABC]} &= -\frac{27}{2}\sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{5\pi}{6}\right) \\
&= -\frac{27}{2}\sin\left(\frac{\pi}{6}\right)\left(-\cos\left(\frac{\pi}{6}\right)\right) \\
&= -\frac{27}{2}\times\frac{1}{2}\times\left(-\frac{\sqrt{3}}{2}\right) \\
&= \frac{27}{8}\sqrt{3}
\end{aligned}$$

9.

Ora,

$$\begin{aligned}
& (\sin x + \cos x)(\sin x - \cos x) = \frac{1}{2} \\
\Leftrightarrow\ & \sin^2 x - \cos^2 x = \frac{1}{2} \\
\Leftrightarrow\ & 1 - \cos^2 x - \cos^2 x = \frac{1}{2} \\
\Leftrightarrow\ & -2\cos^2 x = \frac{1}{2} - 1 \\
\Leftrightarrow\ & 2\cos^2 x = \frac{1}{2} \\
\Leftrightarrow\ & \cos^2 x = \frac{1}{4} \\
\Leftrightarrow\ & \cos x = -\frac{1}{2} \vee \cos x = \frac{1}{2}
\end{aligned}$$

Como $x \in \left]\dfrac{\pi}{2}, \pi\right[$, então, $\cos x < 0$, pelo que $\cos x = -\dfrac{1}{2}$.

Portanto, $x = \pi - \dfrac{\pi}{3} \Leftrightarrow x = \dfrac{2\pi}{3}$.

Assim, vem que $\tan\dfrac{2\pi}{3} = -\tan\dfrac{\pi}{3} = -\sqrt{3}$.

10.

Área sombreada =

= Área do setor circular COB + Área do setor circular AOD + Área do triângulo $[AOB]$

Temos, então, que:

- Área do setor circular $COB = \dfrac{\alpha \times 5^2}{2} = \dfrac{25\alpha}{2}$
- Área do setor circular $AOD = \dfrac{\alpha \times 5^2}{2} = \dfrac{25\alpha}{2}$
- Área do triângulo $[AOB] = \dfrac{\overline{AB} \times h}{2} = \dfrac{10\cos\alpha \times 5 sin\alpha}{2} = 25\cos\alpha\sin\alpha$

Cálculos Auxiliares:

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{h}{5} \Leftrightarrow 5\cos\left(\frac{\pi}{2} - \alpha\right) \Leftrightarrow h = 5\sin\alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \frac{\frac{\overline{AB}}{2}}{5} \Leftrightarrow \frac{\overline{AB}}{2} = 5\sin\left(\frac{\pi}{2} - \alpha\right) \Leftrightarrow \overline{AB} = 10\cos\alpha$$

Assim, vem que:

Área sombreada $= \dfrac{25\alpha}{2} + \dfrac{25\alpha}{2} + 25\cos\alpha\sin\alpha = 25(\alpha + \cos\alpha\sin\alpha)$ $\quad$ *(c.q.m)*

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